The Value of a Controlling Interest in an Expropriated Oil & Gas Company: YPF SA

Patrick L. Anderson
Principal & CEO, Anderson Economic Group LLC

I. Abstract

This technical paper presents an example of the use of different valuation methodologies when applied to the question of the value of the controlling interest that the Spanish firm Repsol held in the integrated oil & gas corporation YPF Sociedad Anonima until its expropriation by the Argentine national government in May 2012. This problem involves two of the signal problems in valuation: the presence of asymmetric risk and real options.

Both traditional and novel methods are used in this analysis, including: standard discounted cash flow and market-multiple models; along with the novel recursive ("value functional") method. A benchmark of market prices around the time of the expropriation is used to evaluate the strength of the valuation methodologies. To make the comparison rigorous, wherever possible the same underlying assumptions and data are used in all methods, with no subjective adjustments.

The following results emerge from this analysis:

1. Traditional valuation methods often produce estimates that differ substantially from market prices when real options or asymmetric risks are present. The magnitude of these estimation errors, as demonstrated here, can easily exceed 50% of the market value of a company.
2. The value functional method can natively incorporate real options and asymmetric risk. Furthermore, the method can produce value estimates that are relatively close to market prices even when real options and asymmetric risks are present.
3. A recursive valuation supports the claim made by Repsol that the value of their expropriated shares in YPF SA was at least $10.5 billion. Traditional methods, unadjusted, produce estimates sig-

nificantly below that figure.

Figure 1: Summary of Estimation Methods for YPF: Recursive vs. Traditional

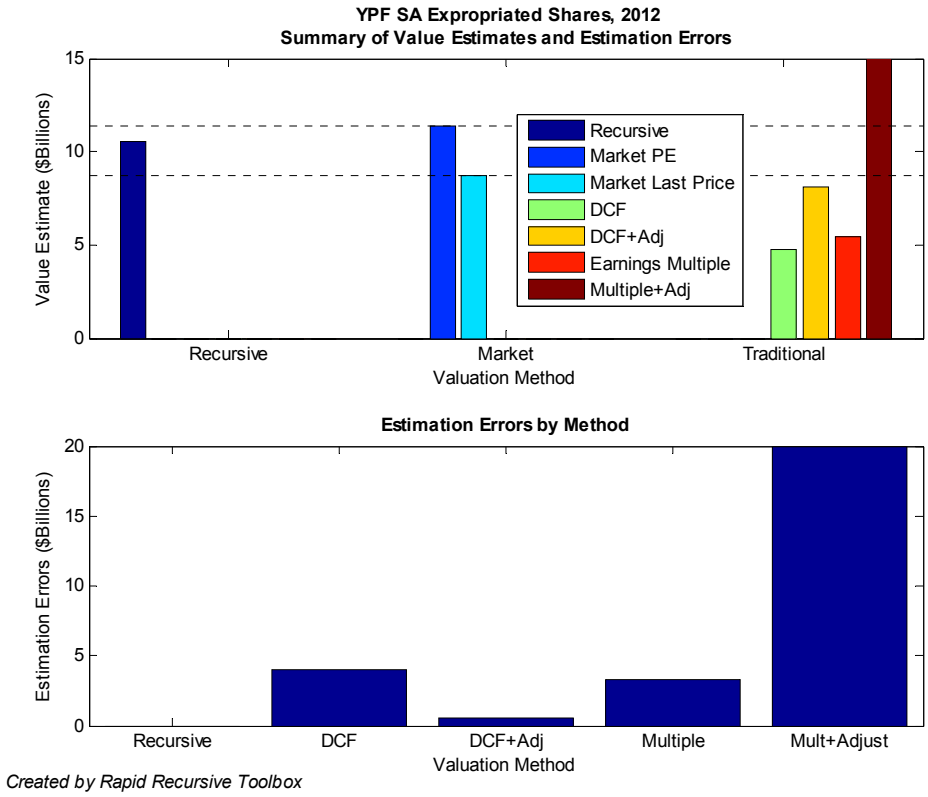


Figure 1 Summarizes the results of a comparison of value estimation methods for the oil & gas company YPF SA in early 2012. Oil & gas companies face both real options and asymmetric risks, which are known to undermine the validity of traditional valuation models.

The value estimates arise from the same baseline assumptions about company revenue, earnings, discount rate, growth rate, and other factors, but different estimation methods. The "market method" estimates are implied by share prices and earnings at that time, and serve as the benchmark for the comparison.

As the graph illustrates traditional methods (such as discounted cash flow and market-multiple) produce estimates that are significantly different from the plausible range of values indicated by actual market prices. Even when subjectively adjusted, they still can be markedly different. The recursive method, in contrast, produced an estimate within the plausible range of values without any subjective adjustments.

II. The Valuation Methodology Question

THE VALUATION QUESTION

This technical paper presents an example of the use of different valuation methodologies when applied to a unique question: the value of the controlling interest in the integrated oil & gas corporation YPF SA held by the Spanish firm Repsol until its expropriation by the Argentine national government in May 2012.

We address this question by using both traditional and novel valuation methods, including:

1. Traditional methods: a standard discounted cash flow model, as well as a market-multiple model.
2. Novel method: the recursive ("value functional") method, for which a recently-introduced commercial software package allows for a straightforward implementation.

For comparison purposes, we also report contemporary analysts' reports on YPF stock, and calculations of contractual damages based on market prices and earnings of the company in the initial months of 2012. These market-based values are then used as the benchmark against which we compare the estimates from traditional and recursive methods.

KEY ISSUES IN VALUATION OF OIL & GAS COMPANIES

The subject company in this case presents two recurring issues in valuation: the presence of asymmetric risks, and real options. These are especially prevalent in industries such as oil & gas, pharmaceuticals, technology, entertainment, and natural resources, and are nearly always present in entrepreneurial and start-up firms.

In the case of YPF SA in the beginning of the year 2012:

- The opportunity to exploit shale oil reserves was a real option of potentially huge value for YPF. However, exercising that option in an aggressive fashion would be very expensive. Strong differences of opinion existed between YPF management (when Repsol was the controlling shareholder) and the Argentine national government on the management decisions related to the exploitation of the shale reserves.[1]
- The potential for big swings in oil prices, technological shifts in the costs of exploiting shale reserves, and the unknowable amount of actual, exploitable shale oil available to YPF combined to produce a rich mixture of asymmetric risks for its shareholders.

The existence of these risks, along with the availability of contemporaneous data, present an excellent factual basis for a methodological comparison among traditional and recursive methods of valuation.

OUTLINE OF TECHNICAL PAPER

The paper is organized as follows:

- The challenges real options and asymmetric risk pose to traditional valuation methodologies is discussed in "Problems with Traditional Valuation Methodologies" on page 9.
- The subject company YPF SA is described in "Information on YPF SA" on page 5.
- The relevant valuation models, including both traditional and the novel recursive method, are described in "Valuation Models Used in this Analysis" on page 11.
- The common data used for all models is described in "Data" on page 8.
- The results are summarized in "Results" on page 14.
- Conclusions regarding the methodology and valuation question are stated in "Conclusions" on page 17,

In addition, we include the following discussions:

- "Appendix I" on page 19.
- "References" on page 19.
- "Limitations" on page 23.

1. News reports (cited in "References" on page 19) confirm that the Argentine national government insisted on aggressive exploitation of the reserves, even at large cost. YPF management wanted to follow a much more measured pace of exploiting the reserves, at a much more modest cost. The company's filings with the SEC confirm that the extent of the shale reserves, and the cost of finding them, refining them, and distributing them to markets, were significant risks for the company.

III. Information on YPF SA

YPF SA

YPF SA is an integrated oil & gas company with extensive reserves in Argentina. It had formerly been a state-owned company, but was privatized in the early 1990's. Its largest shareholder (until the expropriation) had been Repsol, a Spanish energy company. A 51% stake Repsol held in YPF SA was expropriated by the Argentine national government under a law passed in April 2012.[2]

YPF had a history of profitability, and of paying a large share of its earnings out to shareholders in the form of a dividend. However, the Argentine government began to pressure YPF in the years leading up to the expropriation of Repsol's shares to use the cash earnings of the company to aggressively exploit the country's shale oil reserves, rather than pay large dividends.[3] The existence of these reserves had recently been proven, but the extent of them had not.

YPF SA has common stock traded on the Buenos Aires stock exchange, and American Depository Receipts (ADRs) traded on the New York Stock Exchange. Extensive information is available about YPF SA from the following sources, which were used extensively in this analysis:

- Fernandez (2012) contains an excellent summary of YPF's business and relevant events in the years leading up to the expropriation.
- Periodic reports on YPF SA are filed with the United States SEC; selected excerpts from these are cited in this report.
- Yahoo! Finance and other investor-oriented web sites offer access to various reports and summaries of financial information.

2. The Argentine government enacted a "Hydrocarbon Sovereignty" law in April 2012, which (as recorded in YPF SA's Form 6-K filing with the US SEC) included in pertinent part:

   Article 1. - Achieving self-sufficiency in the supply of hydrocarbons as well as in the exploitation, industrialization, transportation and sale of hydrocarbons, is hereby declared a national public interest and a priority for ARGENTINA, with the goal of guaranteeing socially equitable economic development, the creation of jobs, the increase of the competitiveness of various economic sectors and the equitable and sustainable growth of the provinces and regions.
   ...
   Article 7.- For purposes of ensuring the fulfillment of the objectives of this law, FIFTY-ONE PERCENT (51%) of the equity of YPF SOCIEDAD ANÓNIMA, represented by an identical stake of Class D shares held by Repsol YPF S.A., held by its controlled or controlling entities, directly or indirectly, is hereby declared a public interest and subject to expropriation.

See "References" on page 19.

ANALYST REPORTS ON YPF

Fernandez (2012) reports an extensive set of analysts' estimates of the future share price (and therefore implied value) of YPF, published in the months before the expropriation. It is interesting to observe the bunching of these estimates. There is a clear central tendency in the $46-$51 per share interval.We did not attempt to disentangle the analysts' subjective judgements about the stock from their objective analysis.

CONTRACT DAMAGES

The case can also be seen as a classic example of commercial damages. There is no question that the Argentine government expropriated the shares, citing the national interest. YPF's bylaws include sections approved by the Argentinian government that describe compensation to be paid in case of subsequent acquisition of the shares by the same government. Ironically, these were apparently intended to assuage investor worries that YPF would be nationalized in the future.[4]

Fernandez (2012) describes two methods of calculating what might be called liquidated damages under the contract, both of which involve recent share prices (with one involving the share price/net income ratio). He calculates these as $34.30 or $47.30 per share, if the relevant date is April 16, 2012, the day Argentina declared it would expropriate the shares; and higher amounts (of $43.50 and $56.70 per share) if the relevant date is earlier in the year.[5]

3. The company's year-end 2011 Form 20-F reports:

   We have distributed over 85% of our net income attributable to the years 2001 through 2006 in dividends to our shareholders. We have not adopted a formal dividend policy. Any dividend policy adopted will be subject to a number of factors, including our debt service requirements, capital expenditure and investment plans, other cash requirements and such other factors as may be deemed relevant at the time. In the shareholders' agreement entered into by Repsol YPF and Petersen Energía in connection with the Petersen Transaction, they agreed to effect the adoption of a dividend policy under which we would distribute 90% of our net income as dividends, starting with our net income for 2007.... However, following repeated public statements by the Argentine government that YPF had paid too much of its earnings in dividends and requesting the Company to withhold dividend payments for 2010 and 2011 and invest the related funds in exploration and production activities in Argentina, in March 2012, our Board of Directors decided not to pay a cash dividend but rather offer a scrip dividend with respect to 2011 and remaining undistributed prior years' earnings.
4. Fernandez (2012) cites these as Articles 7 and 28 of the bylaws, approved in 1993 at the time of the previous privatization of YPF.

Given that YPF has approximately 393.3 million shares outstanding, these imply a range of possible liquidated contract damages of $6.9 to $11.3 billion, with $8.7 billion being the figure that is derived from the share price in January 2012, and $9.5 billion derived from the price/income ratio from April 16, 2012.[6]

These figures are used as a benchmark for the valuation estimates produced by the various methodologies, as they are based on contemporaneous market prices. Because the Argentine government first signaled that it might expropriate Repsol's interests in January, passed the law doing so in April, and executed the action in May, there is a small range of values that we use as this benchmark.

CLAIMS IN VARIOUS TRIBUNALS

Repsol has sought compensation from the World Bank, through International Center for Settlement of Investment Disputes. The organization agreed to accept the complaint and begin its arbitration. Multiple press reports list the compensation demand of Repsol at $10.5 billion. However, neither the request for arbitration nor the response has yet been made public, and the arbitration body has not yet met.

In addition to this claim, a number of claims have been made in US courts. These suits generally assert that YPF, Repsol, underwriters of securities in these firms, or their officers violated US laws by, among other things, concealing the risks of nationalization, engaging in management actions that improperly increased the dividend to shareholders at the cost of funds needed for exploration, or misrepresenting the company's finances. While multiple press reports assert these claims, the author was able to review only one actual complaint, and did not review any answers to those complaints.[7]

These damages claims have not yet received answers, and the author made no attempt to verify the information in them. Therefore, they are included here only for information.

5. The higher amounts (of $43.50 and $56.70 per share) would arise if the relevant date was January 27, 2012, a date on which the intentions of the Argentine government's leader were made clear to the public.
6. These calculations have been made by the author using figures prepared by Fernandez (2012). It is possible that the reported number of shares or other factors in the calculation are not accurate. In addition to other limitations cited below, the author did not verify the bylaws or audit the figures involved in the calculation.
7. The complaint reviewed is listed in "References" on page 19.

DATA

The following data and assumptions were used in all methods.

TABLE 1: Common Data and Assumptions Used in Analyses

Parameter or Input Variable	Value	Notes
YPF Revenue	$13.185 billion	base year 2011 [Income statement figures from YPF SA Form 20-F Item 3]
YPF Gross Profit	$3.434 billion	base year 2011
YPF Net earnings	$1.232 billion	base year 2011
YPF dividend	$1.232 billion	base year 2011
YPF shares	393.3 million	base year 2011 [Form 20-F item 10]
Repsol share of YPF ownership; Expropriated stake in YPF	57% 51%	YPF SA Form 6-K (filed April 2012)
Expected future dividend payout ratio (or net cash flow to equity as share of net earnings), before expropriation	90%	Author's assumption given historically high dividend payouts; see YPF statement in text
Recent share prices (January and April 2012), as defined in YPF articles of organization for purposes of shareholder compensation in case of nationalization	$43.50; $34.30	Calculated by Fernandez (2012)
Recent analyst estimates of "target" or "fair price" for YPF shares; strong central tendency	$46-$51	Recorded by Fernandez (2012); central tendency identified by author
Implied market price of expropriated stake, based on recent share prices or PE ratios:		
High and low of four possible market method calculations	$6.9 billion; $11.3 billion	Calculated by author from Fernandez (2012) data and interpretation of contract
Median of four calculations	$9.1 billion	
Benchmark range	$8.7-$11.3 billion	Selected by author as portion of range containing the median
Crude oil prices: Average of spot prices, 2011 Average of spot prices, 2012 Spot prices, April 2012	 $94.8, $111.3 $94.1, $111.7 $103.3, $119.8	US EIA data for FOB spot crude, dollars per barrel; US (WTI), European (Brent) (Model is benchmarked on WTI price)
Investor discount rate (*per annum*)	16%	Author's assumption; reflects standard corporate finance model estimate for cyclical industry such as oil & gas; see data sources in Appendix
Trend growth rate in revenue and prices (*per annum*)	2.5%	Author's assumption; assumes slow growth in oil consumption, as a result of higher economic growth and increased efficiency, plus growing oil reserves internationally

Sources: YPF SA filings; Fernandez (2012); US DoE EIA; Author's calculations; Author's estimates. See data sources in Appendix.

IV. Valuation Methodology

PROBLEMS WITH TRADITIONAL VALUATION METHODOLOGIES

Both asymmetric risks and real options are known to render invalid the basic assumptions underlying traditional discounted cash flow ("DCF") valuation methods.[8]

- Traditional DCF models rely on the assumption that the underlying distribution of risks is at least approximated by a smooth, bell-shaped curve of potential outcomes.[9] If this distribution is actually skewed to one side, or contains a significant "black swan" risk of very large magnitude, a single indicator of risk (such as the mean or median of the distribution) is not adequate. For example, when evaluating a company in this industry, one might assume that oil prices over time follow a well-defined stochastic process. However, such a claim cannot be forwarded for shale oil reserves, nor for expropriation risk, nor currency risk.[10] Therefore, companies in this industry are exposed to asymmetric risk that is not represented properly in traditional models.
- Traditional DCF models evaluate one scenario of future economic conditions and management decisions. Therefore, they cannot capture the potential benefits available from changing course in the future, nor distinguish between investment opportunities that have dramatically different potential responses to unforeseen circumstances.

  For example, almost all integrated oil & gas companies have contractual rights to exploit reserves in specific areas of the world, which they may, or may not, choose to exercise in the future. The incentives to exercise these are affected by conditions (such as the market price of oil and gas) that cannot be known at this time.

8. The seminal research establishing this result includes Dixit & Pindyck (1994). Myers (1977) coined the term "real options." Schwartz & Trigeorgis (2001) collect a number of important early papers in the real options literature.
9. The CAPM paradigm and the larger mean-variance framework that underlies much of modern corporate finance are two examples of this. Much of the cost-of-capital data (including the data used in this analysis) available for publicly traded firms in the United States implicitly rely upon some variation of a mean-variance framework.
10. Among other reasons for this are the fact that we have little historical data on the total costs of finding and exploiting shale oil reserves, as well as incomplete information on environmental hazards and long-term impact on areas in which "fracking" is used to extract shale oil. In contrast, we have approximately 100 years of data on oil prices. See "References" on page 19 for data sources used in this paper.

Only one scenario for the exploitation of these reserves can be incorporated into a DCF model.[11]

COMMON PRACTICE OF "ADJUSTING" ESTIMATES

As a result of the deficiencies noted above, it is common for analysts to "adjust" their assumptions used in a DCF analysis. This allows for factors that cannot be captured by the underlying methodology to be included in the valuation estimate. It also allows for a subjective estimate (or an estimate made from an entirely different method or set of data) to determine the estimate, even if it is attributed to a discounted cash flow analysis.

THE RECURSIVE METHOD

A novel alternative to traditional DCF analysis is known as a "recursive" or "value functional" method.[12]

The recursive method involves casting the objective of the manager or investor as the optimization of the value of the company or investment, across all available actions. Here "value" is defined recursively, as the sum of current earnings and discounted future value. This turns the multi-period forecasting problem of the DCF method into a series of two-period optimization problems.

The key mathematical equation within the method is known as a value functional equation, where a "functional" can be understood as a function of other functions.[13] Anderson (2013) lays out a value functional approach to the valuation of operating businesses, which was implemented in the manner described in detail in "Appendix II: Recursive Model" on page 21.

11. Of course, one can create two or three or several DCF models. However, doing this properly requires adjusting not only the revenue, but also the implied management policies and therefore costs as well. This rapidly becomes very complicated if done in spreadsheet software, and relies on manual adjustments of potentially dozens of lines in each of several income statements; and then the knotty task of deciding which of these resulting scenarios should be the basis for the investment decision.
12. Depending on the academic field, recursive methods are sometimes called impulse control, stochastic control, or dynamic programming. Anderson (2013) describes the method with regard to businesses as "value functional," and compares it with other theories of value arising from Economics, Finance, and traditional usage.
13. The underlying mathematical theory is due to Bellman (1957), and was further developed by Stokey & Lucas (1989).
The method applied to business valuation was first outlined in Anderson (2004). See Anderson (2013) for an extended exposition on the underlying theory and practical examples of valuation using actual companies.

BENEFITS OF THE RECURSIVE METHOD

The recursive method has significant advantages for problems of the type presented here. In particular, the recursive or value functional method natively handles both asymmetric risks and real options. In particular:

- Asymmetric risk can be handled by explicitly incorporating transition probabilities (reflecting the odds of moving from one set of conditions to another) that are not symmetric. This can include unusual, but very damaging, "black swan" events.
- Real options can be handled explicitly by incorporating the possible exercise of management flexibility. Transaction costs can also be included.

Also, the value functional method produces a value estimate for each set of conditions (or "states"), as well as a value-maximizing policy. Because the management decision to exploit shale reserves aggressively, or slowly, was a contentious issue, the use of the recursive model could provide some insight into this decision.

DIFFICULTIES OF USING A RECURSIVE APPROACH

There are significant difficulties to implementing a recursive or value functional approach to decision problems of this type. First, simply formulating mathematically and solving a value functional equation can be a daunting task, particularly since some such equations do not have a solution.[14] Furthermore, until recently no commercially available software would formulate and solve value functional equations for common decision problems. However, we utilized a recently-introduced commercial software product that facilitates the composition, error-checking, formulation, and solution of the underlying value functional equation.[15] Finally, using the recursive method requires the use of more information than with traditional models, particularly information related to asymmetric risks, and the availability, costs, and consequences of possible management policies.

VALUATION MODELS USED IN THIS ANALYSIS

The following valuation models (with the key underlying algorithm for each) were used in this analysis:

14. Stokey & Lucas (1989) provide existence theories for certain types of problems. Anderson (2013) describes a set of propositions for business and investment problems that have solutions, and for which solutions can be found. Both these references describe at least two iterative algorithms that are known to solve, at least eventually, value functional problems that meet theoretical requirements for a solvable problem.
15. The software used in this analysis includes Matlab and the Rapid Recursive Toolbox, both of which are listed in "References" on page 19.

1. Traditional DCF (NPV)
   Consistent with the theoretical underpinnings of this method,[16] and the goal of producing an unadjusted estimate that could be compared across methods, the net present value of a perpetual stream of distributed earnings to shareholders, based on current earnings, was used as the DCF estimate.
   A number of assumptions used in this method were adopted throughout the other methods, including: an underlying growth trend in revenue and costs in the industry; a discount rate for investors; current revenue, earnings, and dividend payout ratio for YPF.
2. Subjective DCF: NPV Plus Judgement on Real Options ("XNPV")
   An "expanded net present value" estimate was created by taking the traditional (unadjusted) DCF estimate, and then subjectively adding an amount for the real option.
   The subjective adjustment used here is subject to the same criticism as any other subjective judgment: it allows the analyst to, consciously or unconsciously, "adjust" a calculated amount to an amount that could be nearly any possible number.
3. Traditional Market Multiple (Industry P/E ratio)
   A market multiple of earnings was used, with "integrated oil & gas" used as the industry and the price/earnings ratio as the multiple, to create this estimate. The 2011 fiscal years earnings of YPF was used as the basis for the multiple.
4. Subjective Market Multiple (Selected P/E)
   Similar to the DCF-plus-adjustment method mentioned above, a common subjective adjustment to a market multiple is to choose the multiple from the high or low end of the comparison group of companies. We considered this as another example of a subjective adjustment method.
5. Recursive Model (Value Functional)
   A recursive model was implemented using the framework described by Anderson (2013), in which the value of the company given a set of market and economic conditions is determined by the variable listed in Equation (EQ 1) and the solution to the value functional equation shown in Equation (EQ 2).
   Here, the "reward" function is the distributed profits of the company, just as in the classic dividend discount model of Finance.[17] For comparison purposes, the basic DCF net present value equa-

16. See Anderson (2013) for a historical discussion of this method, including the attribution to Joel Dean's popularization of capital budgeting theory and the Modigliani-Miller papers in the 1950s and 1960s. Anderson notes that a capitalization of current earnings is the value assumption used within the famous Modigliani-Miller propositions. See also Rubinstein (2006).

tion is shown, using the same variables where possible, in Equation (EQ 3).

Variables that were used in the DCF model were also used in the recursive model, including the discount and growth rates, recent revenue and earnings of YPF, and their dividend payout ratio. The recursive model natively incorporated asymmetric risks and costs of exploitation that were not used in the DCF estimate. These were separately estimated.

Note on benchmarking of DCF and Recursive Models. To ensure a strong basis for methodological comparison, the DCF and recursive model were bench marked on the following variables: recent-year's earnings, dividend payout ratio, trend growth rates, discount rates for investors, market and industry conditions of oil price and extent of shale reserves. This benchmarking means that the single scenario used in the DCF estimate begins with the same assumptions as the "92 NS" state in the recursive method. This state represents a world oil price (WTI) of approximately $92 per barrel (approximately that during the years 2011 and 2012, although it was $103 in April of 2012), and no additional shale reserves beyond those already exploited by YPF at the end of 2011 and reported in their 2011 annual report.

Any difference in the valuation estimate between the DCF and recursive methods (in state "92 NS") are therefore due to methodological differences or additional information used in one or both of the models.

SOFTWARE

Standard spreadsheet software was adequate to calculate the results for all the traditional models.[18] For the recursive method, the mathematical vector-processing software MATLAB® was used as a general platform. The Rapid Recursive Toolbox® was then used to compose, error-check, formulated, and solve the underlying recursive model.[19] Results were calculated using routines developed by the author as well as available routines from the software mentioned above.

17. In more flexible (or less rigorous) form, the "net cash flow to equity" method that is a workhorse discounted cash flow model is quite similar.
18. Some of the underlying parameters used in these models, particularly discount rates, were estimated by others using statistical software.
19. "Matlab" and "Rapid Recursive toolbox" are products of The Mathworks and Supported Intelligence LLC, respectively. See "References" on page 19 for a listing of software providers.

V. Results

PRESENTATION OF UNADJUSTED RESULTS

Using wherever possible the same assumptions and base data, we estimate the value of Repsol's YPF shares using multiple methods. We then compare them to see which methods come closer to the actual market price (and claims of Repsol at the time of the expropriation).

Note that we use unadjusted estimates for both traditional and recursive methods, generated from the same set of assumptions, as the primary basis for the methodological comparison. This allows a straightforward comparison of the methods, rather than an assessment of the subjective judgment of the analyst. We also present separately results that incorporate a subjective adjustment that is explicitly identified.

RESULTS OF TRADITIONAL METHODS

The baseline data used to estimate results for all methods is summarized in Table 1, "Common Data and Assumptions Used in Analyses," on page 8. To recap the most important of these data:

- Net earnings and dividends of YPF SA were approximately $1.3 billion in 2011, on revenue of over $13 billion.
- World oil prices had been relatively stable at around $94/barrel in the U.S. (for West Texas Intermediate crude).
- Market prices and YPF earnings in January and April 2012 suggested Repsol's 51% stake in YPF SA was worth between $8.7 and $11.3 billion, as discussed under "Contract Damages" on page 6.

The benchmark for our valuation estimates, then is an amount approximately between $9 billion and $11 billion, as indicated by market prices. These market prices reflect investor's knowledge of the world oil market, YPF's conventional and shale oil reserves, YPF's business and ability to earn income, and (with increasing specificity beginning in April 2012) Argentina's efforts to pressure the company to undertake an aggressive and expensive shale oil exploitation effort.

Results: Traditional DCF. A traditional discounted cash flow method, without overwhelming subjective adjustments, involves projecting a single scenario for the future earnings of the company and taking the net present value of these expected earnings. In this case, we projected continued growth of the company's profits in the future, and capitalized it at the cost-of-capital suggested by an analysis of the historic risk and return statistics of the oil & gas industry.

This naive (unadjusted) method substantially underestimates the value of the company as of May 2012, suggesting an intrinsic value of approximately $4.7 B.

Results: Traditional Market-Multiple. Similarly, a market-multiple method, using a multiple-of-earnings typical of the industry, would also suggest the company was undervalued by the stock market as of May 2012. An industry market multiple applied to actual YPF earnings suggests an intrinsic value of approximately $5.5 billion.

"Adjusting" Traditional Methods. Making significant subjective adjustments can move an estimate closer to the market price. In this case, we adjusted the straightforward estimates from traditional methods as follows:

- We used a high (rather than median) market multiple for earnings. Very large oil companies with substantial reserves were trading at 100x earnings, where the industry average was less than 7x earnings. We made a subjective adjustment to the ratio and used 1/2 the high-side multiple, motivating this by the conjecture that investors were paying higher multiples for certain companies because they had large reserves that were not yet exploited.
- We took an analyst's estimate recorded in Fernandez (2012) that the value of the un-exploited shale oil reserves was about $6 billion. It is not known how the analyst arrived at this figure, and other conjectures about the value ranged well above $20 billion. Adding a real option value estimate to an estimate of the value of the earning from current operations is sometimes called "expanded net present value" or "XNPV."

These adjustments are, of course, both subjective and motivated by a desire to explain the large difference between the estimate using a straightforward method and the actual price.

RESULTS OF RECURSIVE METHOD

Results: Recursive Method. We used the same basic assumptions about underlying earnings growth in the industry and discount rate for investors as with the traditional methods. We also used additional information regarding asymmetric risks and real options that the recursive method natively incorporates, including:

- Public knowledge, available at the beginning of January 2012, that Argentina was pressuring YPF to make very large expenditures on shale exploitation. This was implemented in the recursive model as a reduction in the dividend available to investors in the future.
- Probabilities that share reserves would be found and be exploitable, with the probability increasing if the company spent additional money on exploration.

- Probabilities of oil prices going up, down, or remaining the same. Although we could have used an asymmetric distribution here, we implicitly assumed that prices would fluctuate around $105/barrel in the U.S., plus an inflationary adjustment in the future.

No subjective adjustments were made to the estimate.

The results for the recursive method suggest the intrinsic value of Repsol's investment, at the level of exploitation of shale reserves and market price of oil that prevailed in early 2012, was approximately $10.5 billion.

We also completed a recursive valuation of an unconstrained YPF, as a comparison. Assuming the (counter factual) case that Argentina was not pressuring YPF to spend additional amounts on exploration of shale, the recursive method would estimate the value of the firm at that time at $14.7 billion. This is higher than the market price indicated at the time. Together with the value estimated for YPF operating under constraints, this suggests that investors recognized Argentina's interference with the company (which did not yet result in expropriation) as reducing its value.

The summary results are shown in Table 2 on page 16.

TABLE 2: Comparison of Results: Value Estimates Using Different Methods

Method	Value of Shares	Notes
Income method (DCF)	$4.7 billion	Using current earnings (from previous fiscal year)
Income method plus subjective adjustment by adding real option value estimate for rights to exploit shale (XNPV)	$8.1 billion	Subjective adjustment for real option: NPV of earnings from new shale operations planned by management at the time
Market-multiple method (P/E ratio) for oil & gas industry, unadjusted	$5.5 billion	Using standard multiple of earnings for industry
Market-multiple method, with subjective adjustment by use of higher multiple	$31.3+ billion	Using half of high multiple, assuming high shale reserves
Recursive method, constrained YPF management	$10.5 billion	Incorporating same assumptions as with Income method, plus additional parameters unique to this method.
Recursive method, for unconstrained YPF management	$14.7 billion	Assuming that YPF was not constrained by Argentina
Implied by market prices, January-April 2012	$8.7-11.3 billion	See discussion of contract damages in text.
Repsol damage claim	$10.5 billion	
Source: Anderson Economic Group LLC research; author's analysis		

A graphical exhibit of these results is “Comparative Results Using Different Methods” on page 18. A summary exhibit for selected methods (with estimation errors calculated) is “Summary of Estimation Methods for YPF: Recursive vs. Traditional” on page 2.

VI. Conclusions

The results demonstrate that the theoretical superiority of the recursive method can result in practical improvements in valuation, when compared with other methods. Furthermore, the results demonstrate that the results can be substantially more accurate when real options and asymmetric risks are present.

Part of the reason for the improved results is the additional information captured by the model. In particular, the recursive method allowed, in this case, for the exercise of the real option to delay or accelerate exploitation of possible shale oil reserves. This real option, which cannot be natively handled correctly in a traditional DCF model, is a significant source of value to companies that have potentially valuable, but expensive to exploit, opportunities. This is often the case for companies involved in real estate, natural resources, rights to intellectual property, pharmaceuticals, and new technology.

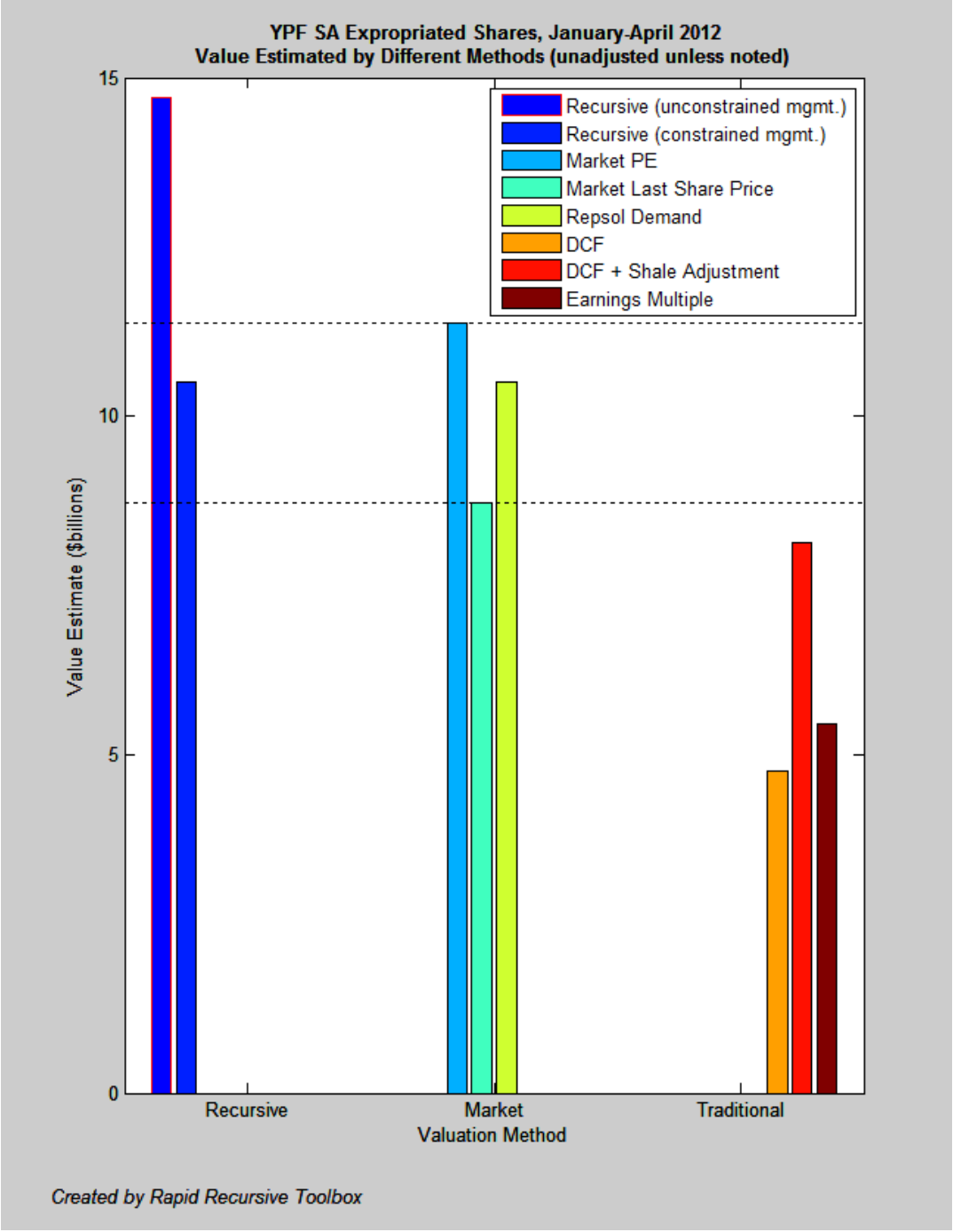
The recursive method, while more powerful, does require additional information. In this case, the additional information was readily at hand. Even subjective estimates of such parameters as the risk that once exploited, the shale reserves would fall add valuable information to the model.

A second implication of the analysis is that the recursive-method valuation software recently introduced does have the capability to model complex valuation questions, and do so in a manner that can provide significantly better results (before any subjective adjustments) than competing traditional models.

A third implication of the results relates to the claim of Repsol that it was deprived of over $10 billion in stockholder value by the expropriation of its shares by the Argentine government. This claim cannot be fully evaluated at this time, as the tribunal that has accepted it (an arm of the World Bank) has not yet heard the case, and neither Repsol’s claim nor Argentina’s response is available at this time. However, the results of the recursive method support the claim that the company did indeed have a market value that implies Repsol’s shares

were worth, at least as an order-of-magnitude estimate, around $10 billion. In contrast, looking only at an unadjusted DCF analysis suggests the value of the expropriated shares was much lower.

Figure 2: Comparative Results Using Different Methods



VII. Appendix I

REFERENCES

SOFTWARE

Matlab® is a product of The Mathworks; http://www.mathworks.com.

The Rapid Recursive® Toolbox is a product of Supported Intelligence LLC; http://www.supportedintelligence.com.

SCHOLARLY PUBLICATIONS

Anderson, Patrick L. (2004) *Business Economics and Finance.* CRC Press.

Anderson, Patrick L. (2009). "The Value of Private Firms in The United States," *Business Economics,* vol. 44, pp. 87–108.

Anderson, Patrick L., (2013) *Economics of Business Valuation.* Stanford University Press.

Bellman, Richard. (1957) *Dynamic Programming.* Princeton University Press; reissued by Dover, 2003.

Dixit, Avanish, & Robert S. Pindyck (1994). *Investment Under Uncertainty.* Princeton University Press.

Ljungqvist, Lars, & Thomas J. Sargent (2000 [2005] [2012]). *Recursive Macroeconomic Theory.* MIT Press.

Modigliani, Franco & M. Miller, (1958). "The Cost of Capital, Corporation Finance and The Theory of Investment" *American Economic Review*, vol. 48, pp. 261-97.

Myers, Stewart C. (1977). "Determinants of Corporate Borrowing," *Journal of Financial Economics*, no. 5, November 1977, pp. 147-175.

Rubinstein, Mark A. (2006). *A History of the Theory of Investments: My Annotated Bibliography.* Wiley Finance.

Schwartz, E., & Trigeorgis, L. (editors.) (2001). *Real Options and Investment Under Uncertainty: Classical Readings and Recent Contributions.* MIT Press.

Stokey, Nancy & Lucas, Jr., Robert E. (with Prescott, E.C.) (1989). *Recursive Methods in Economic Dynamics.* Harvard University Press.

MARKET, INDUSTRY AND FINANCE DATA

Historical stock prices and analyst coverage: Yahoo! Finance

Historical risk premium, rate-of-return, and variance-of-earnings data for publicly-traded companies: Aswath Damodaran; found at http://pages.stern.nyu.edu/~adamodar.

World crude oil prices: US Department of Energy, EIA; found at http://www.eia.gov.

Energy market data: British Petroleum, *Statistical Review of World Energy 2012;* found at http://www.bp.com.

SUBJECT COMPANY AND RELATED DATA

YPF SA form 6-K filed with the US SEC, April 19, 2012, regarding the expropriation under Argentine law of Repsol's 51% stake in YPF.

YPF filings in December 2012 through February 2013 with the US SEC, including various forms 6-K for YPF ADR (American Depository Receipts) regarding claims made in US and other judicial or arbitration bodies regarding the actions of its underwriters, officers, or the Argentine government.

YPF Form 20-F (annual report) for the year ending December 31, 2011, filed with the US SEC.

Fernandez, Pablo, (Nov. 2012 [2013]) "The Value of an Expropriated Oil Company: The case of YPF and Repsol in Argentina," IESE Business School Working paper. Available at SSRN at abstract number 2176728. Revised version dated January 2013.

Complaint filed in *Monroe County Employees' Retirement System v. YPF Sociedad Anonima et al*, New York (SD), 2013; case:13-cv-00842-SAS.

Case detail summary in *Repsol, S.A. and Repsol Butano, S.A. v. Argentine Republic*; World Bank ICSID, case ARB/12/38.

NEWS REPORTS

Various news reports on the expropriation in April and May, 2012; including those published by *Fox News, CNN Money, Bloomberg,* and *Forbes,* with substantially the same content.

Various news reports on complaints filed in February 2013 in lawsuits against YPF, Repsol, their underwriters and officers; including those in *Reuters, Bloomberg,* and *Bloomberg Business Week,* with substantially the same content.

VIII. Appendix II: Recursive Model

VALUE FUNCTIONAL EQUATION METHODOLOGY

The value functional ("recursive") method for use in investment and business valuation analysis, as outlined in Anderson (2013), involves the equations, variables, and conventions described below.

Functional Equation Variables **(EQ 1)**

$V(s,t) =$ value at time t given state s

$f(s,x) \equiv$ reward function given state and action

$g(s,x) \equiv$ transition function

$t = 0,...,T$ time index

$s_t = s_0,...,s_T$ state variables

$x_i = x_0,...,x_M$ action or control variables

$\beta =$ discount factor

$\varepsilon =$ random error term

Value Functional Equation **(EQ 2)**

$$V(s_t) = \max_{x \in \Gamma} \left\{ f(s,x) + \beta E\left[V(s_{t+1}) \right] \right\};$$

where:

$s =$ state;

$x =$ action; $\Gamma =$ feasible set of actions.

Net Present Value Equation, Discounted Cash Flow Method **(EQ 3)**

$$NPV = \sum_{t=0}^{T} \beta^t C_t$$

where:

$t =$ time index,

β=discount factor; $0 < \beta < 1$;

$C_t =$ cash flow at time t.

Implicit Assumption in DCF versus Recursive Methods. Note that, while the DCF method implicitly assumes that investors passively receive future earnings from an asset, the recursive method assumes that they actively maximize value even as conditions change.

Implementation of Recursive Model. Key elements in the value functional model used in the analysis are further explained below:

- The transition function captures the uncertainty about future conditions, including the asymmetric risks involved in the exploitation of shale reserves and the uncertain movements of oil prices.
- The states represent combinations of two varying market or industry conditions: oil prices, and shale reserves available to YPF. These were coded as both world oil prices (92, 100, and 110 dollars) and gradations of additional shale reserves exploited by YPF (none, a large increase, and a very large increase), with 9 possible combinations of these conditions.
- The actions represent the key choices to exploit the reserves, ranging from a measured pace of exploitation, which was favored by Repsol, to a hurried pace, favored by the Argentine government.
- In the constrained-management version of the recursive model, YPF investors lose a portion of their dividend as it is forced to be spent on shale exploitation activities or other expenditures at the behest of the national government. The lost amount is less than the amount of a full shale exploration program, reflecting an assumption that some fraction of the government-demanded expenditures did indeed add value to the company.
- The model was created as a discrete-time, discrete-state Markov decision problem.[20] Thus, the transition and reward functions were represented by matrices, with elements that corresponded to combinations of states and actions. The benchmark state-action combination (in which the market conditions and management actions approximated those in late 2011, before public notice of an impending expropriation) is an oil price of approximately $92 (close to the average of the calendar year 2011), and no additional shale reserves beyond those announced by YPF in their year-end 2011 annual report.
- The discount factor used in Equation (EQ 2) (represented by the greek letter *beta*) is a net discount factor that combines the effect of both trend growth rates in revenue and prices and the discount rate applied by investors or managers on the investment itself.[21] Thus, it captures the expectation of growth and inflation, as well as discounting for time and risk.

The model was composed, formulated into a mathematical equation, and solved using the Rapid Recursive® toolbox and Matlab® software listed in “Software” on page 19.

20. The “Markov” property means that all the relevant information available for forecasting future values of key variables is captured in their values for the current period. The underlying equation for a Markov Decision Problem is a type of value functional equation that is particularly suited for business valuation and investment analysis.

EXTENSIONS OF CURRENT PROBLEM

An obvious extension would be to include an explicit risk of expropriation into the recursive model. The recursive method used recognized the burden of interference, but not that of expropriation. However, it is likely that stock market investors were as surprised as Repsol and other national governments when Argentina actually expropriated the shares, and the valuation benchmark is based on prices and earnings from before the expropriation occurred.

Other improvements or extensions include:

- Improving the estimates for added costs, and possible increased earnings, for different levels of shale reserves.
- Improving the estimates for future oil prices.
- Decomposing the income statement and dividend policy of YPF in a better fashion.

It is worth noting that, because the same basic data were used in all methods, it is unlikely that improving any of the listed areas above would significantly change the results of the methodology comparison.

Additional uses. The results confirm that a recursive model can be deployed in practical investment and valuation work. They also suggest that companies and investments that involve real options or asymmetric risks are particularly good targets.

LIMITATIONS

This report is subject to the following limitations:

- The primary purpose of this analysis is to compare valuation methodologies, not to estimate damages in any of the cases involving YPF, nor to provide investment advice.
- All value estimates are based on expectations about future events, which are inherently uncertain. Therefore, all value estimates are "forward looking statements" and cannot be expected to reliably predict the future.
- The author did not examine or audit the books and records of YPF SA, nor review its management actions, did not independently review the rights of its shareholders under its charter and Argen-

21. Here, $beta=(1+g)/(1+d)$, where g, d are assumptions about trend growth rates and investor discount rates, respectively. The use of the same factor in the DCF relation shown in Equation (EQ 3) is ambiguous, as the manner in which future cash flows have been forecasted is not specified. However, for the methodological comparison used in this paper, the same growth and discount factor assumptions were used in both models.
(Note that the *beta* here should not be confused with the *beta* commonly used in the CAPM, or as a shorthand for variation in stock prices relative to the stock market as a whole.)

tine law, and had no access to confidential data from the company or any of its shareholders. Readers seeking such a review or audit should therefore seek other authorities.

- The author did not evaluate any of the legal claims made by various claimants in cases involving YPF, beyond the simple reporting of some aspects of those claims here.
- Given the complexity of the calculations involved in multiple methods, and the amount of data required, it is possible that a data, calculation, or other error in incorporated in this report. Readers that observe any errors can contact the author at the author contact address listed below, noting the error and suggesting a correction.[22]
- The author may revise this report in the future if new information becomes available.

22. Contact the author at Anderson Economic Group LLC, Chicago Illinois, by mail at the address listed on the company website (at http://www.andersoneconomicgroup.com), or via email at panderson@andersoneconomicgroup.com.

